Date: 16/9/2015 TEST -4 PHYSICAL SCIENCES Time: 2 Hrs

CLASSICAL MECHANICS M.M: 60

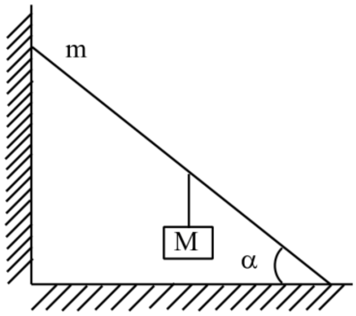
1. A particle is moved quasistatically from point (-3,0) to (3,0), along a path $y = x^2 - 9$, in an external force field given by $\vec{F} = y\hat{i} + 3y\hat{j}$. All quantities are in SI units, the magnitude of the work done on the particle is given by

   (1) 36 J (2) 18J (3) 9J (4) 0

2. A ladder of mass *m* is leaning against a wall, as shown in the figure given below. A mass *M* is hung from the middle of the ladder. The wall and the ground have a coefficient of friction μ. What is the value of the angle α below which the ladder cannot remain in static equilibrium?

   (1) $\tan\alpha = \frac{2\mu}{1-\mu^2}$ (2) $\tan\alpha = \frac{2\mu}{\frac{m}{M} - \mu^2}$

   (3) $\tan\alpha = \frac{2\mu}{\frac{M}{m} - \mu^2}$ (4) $\cot\alpha = \frac{2\mu}{1-\mu^2}$



3. A simple pendulum, consisting of a small ball of mass *m* attached to a massless string hanging vertically from the ceiling, is oscillation with an amplitude such that the maximum tension in the string is related to the minimum tension $yT_{\max} = 2T_{\min}$. What is the value of this maximum tension in the string?

   (1) $\frac{mg}{2}$ (2) $\frac{3mg}{2}$ (3) $\frac{3mg}{4}$ (4) $3mg$

4. The trajectory of a particle of mass m is described in the cylindrical polar coordinates by $\dot{z} = 0, \dot{\phi} = \omega$ and $r(t) = r_0 \sinh(\omega t)$, where ω and r_0 are constant. The radial force is:

   (1) $mr_0^2\omega^2 \sinh(\omega t)$ (2) $mr_0\omega^2 \sinh(\omega t)$

   (3) $mr_0\omega^2 \cosh(\omega t)$ (4) 0

5. A vertical tunnel is dug up through earth passing through the centre. A particle of mass m is thrown inside the tunnel. How should the density vary with radius R inside the earth if the particle executes perfect simple harmonic motion?

   (1) $\rho(R) \propto \frac{1}{R}$ (2) $\rho(R) = \text{constant}$ (3) $\rho(R) = \text{constant } or\ \rho(r) \propto \frac{1}{R^2}$

   (4) $\rho(R) \propto \frac{1}{R^2}$

6. An object is made from a thin wire and is shaped like a square with *a* side L and a total mass *m*. What is the moment of inertia of this object around an axis that passes through the centre of the square and is perpendicular to it?

(1) $\frac{7}{6}mL^2$ (2) $\frac{1}{2}mL^2$ (3) $\frac{4}{3}mL^2$ (4) $\frac{16}{3}mL^2$

7. Moment of inertia of a solid cylinder of mass *m*, height *h* and radius r about an axis passing through its centre of mass and perpendicular to its axis of symmetry is:

(1) $\frac{mr^2}{4}+\frac{mh^2}{12}$ (2) $\frac{mr^2}{2}+\frac{mh^2}{8}$ (3) $\frac{mr^2}{2}+\frac{mh^2}{12}$ (4) $\frac{mr^2}{2}+\frac{mh^2}{4}$

8. A thin massless rod of length 2ℓ has equal point masses *m* attached at its ends. The rod is rotating about an axis passing through its centre and making angle θ with it. The magnitude of the rate of change of its angular momentum is:

(1) $2m\ell^2\omega^2\sin\theta\cos\theta$ (2) $2m\ell^2\omega^2\sin\theta$

(3) $2m\ell^2\omega^2\sin^2\theta$ (4) $2m\ell^2\omega^2\cos^2\theta$

9. A circular plate form is rotating with a uniform angular speed ω counter clockwise about an axis passing through its centre and perpendicular to its plane. A person of mass m walks radially inward with a uniform speed v on the plate form. The magnitude and the direction of the Coriolis force (with respect to the direction along which the person walks) is

(1) $2m\omega v$ toward his left (2) $2m\omega v$ towards front

(3) $2m\omega v$ toward his right (4) $2m\omega v$ towards his back

10. A solid sphere of mass m and radius *a* is rolling with a linear speed v on a flat surface with slipping. The magnitude of the angular momentum of the sphere with respect to a point along the path of the sphere on the surface is:

(1) $\frac{2}{5}mav$ (2) $\frac{7}{5}mav$ (3) mav (4) $\frac{3}{2}mav$

11. A hollow sphere of moment of inertia $\frac{2}{3}mR^2$ and a thin loop of moment of inertia mR^2 roll without slipping down an inclined plane. The ratio of their times of arrival T_{sphere}/T_{loop} at the bottom of the incline is given by

(1) $\sqrt{\frac{1}{2}}$ (2) $\frac{1}{2}$ (3) $\frac{3}{\sqrt{5}}$ (4) $\sqrt{\frac{5}{6}}$

12. A uniform rod of mass m and length ℓ is hinged at one of its end O and is hanging vertically. It is hit at its midpoint with a very short duration impulse *J* so that it starts rotating about O. The magnitude of the horizontal impulse applied by O on the rod is

(1) $m\sqrt{g\ell}$ (2) $2m\sqrt{\frac{2g\ell}{3}}$ (c) $2m\sqrt{\frac{3g\ell}{2}}$ (4) $m\sqrt{\frac{3g\ell}{2}}$

13. Particles A and B of mass *m* and particle C of mass *m* are placed along *x*-axis in order. Particle A is given momentum $mv\hat{i}$, consequently there are two perfectly inelastic collisions. The energy loss is $\frac{7}{8}$ of the initial energy (ignore the loss due to friction), the m is:

(1) 2m (2) 4m (3) 6m (4) 8m

14. A proton of mass *m* collides with a particle of unknown mass at rest. After collision, the proton and the unknown particle are seen moving at an angle of 90^o with respect to each other. The mass of the unknown particle is:

(1) $\frac{m}{2}$ (2) m (3) $\frac{m}{\sqrt{3}}$ (4) $2m$

15. A chain of mass *m* and length *L* is hanging vertically over a table, with its lowest point touching the surface of the table. It is released and it falls on the table completely inelastically. How much time does it take for the chain to tall completely on the table?

(1) $\sqrt{\frac{L}{2g}}$ (2) $\sqrt{\frac{2L}{3g}}$ (3) $\sqrt{\frac{L}{g}}$ (4) $\sqrt{\frac{2L}{g}}$

16. The of a free particle in spherical polar coordinates is L. The conserved quantity is $\left\{L = L = \left(r, \theta, \phi, \dot{r}, \dot{\theta}, \dot{\phi}\right)\right\}$

(1) Energy and p_r (2) Energy only

(3) Energy and p_θ (4) p_θ only

17. A particle of mass *m* is falling vertically under the gravity (other force are neglected), the degree of freedom of the particle is:

(1) 1 (2) 2 (3) 3 (4) 0

18. The Lagrangian of the charged particle of mass *m*, charge *q* in electromagnetic field is given as:

(1) $\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-q\left(\phi+\vec{v}.\vec{A}\right)$ (2) $\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+q\left(\phi+\vec{v}.A\right)$

(3) $\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\left(-q\phi+\vec{v}.\vec{A}\right)$ (4) $\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-q\phi$

19. A particle of mass *m* is moving under the point $V = \frac{1}{2}x^2 + \frac{1}{2}y^2 + k\dot{x}\dot{y}\dot{z}$. The conserved quantities are:

(1) z-component of linear and angular momentum only
(2) energy and z-components of linear and angular, momenta
(3) z-component of linear momentum only
(4) z-component of angular momentum only

20. A particle is moving under the action of a generalized potential $V\left(x,\dot{x}\right) = \frac{1+\dot{x}}{x^2}$. The magnitude of generalized force is

(1) $\frac{2\left(1+\dot{x}\right)}{x^3}$ (2) $\frac{2\left(1-x\right)}{x^3}$ (3) $\frac{2}{x^3}$ (4) $\frac{\dot{x}}{x^3}$

21. The Lagrangian of a particle of mass m moving in one dimension is

$L = \exp\left[\gamma t\left(\frac{m\dot{q}^2}{2} - kq^2\right)\right]$ Where, γ and k are positive constants. The equation of motion is:

(1) $\ddot{q}+\gamma\dot{q}=0$ (2) $\ddot{q}+\frac{k}{m}q=0$ (3) $\ddot{q}-\gamma\dot{q}+\frac{k}{m}q=0$ (4) $\ddot{q}+\gamma\dot{q}+\frac{k}{m}q=0$

22. The Lagrangian of a system is given by $L = \frac{1}{2}\dot{x}^2 + x\dot{x} - \frac{1}{2}x^2$. It describes the motion of

(1) A harmonic oscillator (2) a damped oscillator
(3) An harmonic oscillator (4) system with unbound motion

23. If a particle moves outwards in a plane along a curved trajectory described by $r = a\theta, \theta = \omega t$, where a and ω are constants, then its
(1) Kinetic energy is conserved
(2) Angular momentum is conserved
(3) Total momentum is conserved
(4) Radial momentum is conserved

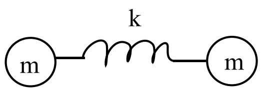
24. A mass m is attached to one end of a spring of spring constant k. Other end of the spring is held fixed. The motion of mass (along the spring) is described by the Lagrangian:
(1) $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$ (2) $\frac{1}{2}m\dot{x}^2 - kx^2$
(3) $\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + dx + \omega\dot{x}t$ (4) $\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + \propto \dot{x}\sin\omega t + \propto \omega x \sin\omega t$
Where $\propto$ and ω are constants with suitable dimensions.

25. A particle is moving under the influence of the potential $-\frac{k}{r}$, where k is a constant. If L_x, L_y and L_z denote the angular momenta in three directions, then
(1) L_x and energy are conserved
(2) L_x, L_y and energy are conserved
(3) L_x, L_y, L_z are conserved
(4) L_x, L_y, l_z and energy are conserved

26. Lagrangian of a particle is given as
$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{1}{2}\left(x^2 + y^2\right)$. The correct statement is
(1) Force along x-direction is zero
(2) Force along y-direction is zero
(3) Force along z-direction is constant and torque about z-axis is zero
(4) Force along z-direction is zero and torque about z-axis is zero

27. A particle of mass m is subjected to the force $\vec{F} = F_0 \sin\omega t\hat{i}$. The conserved quantities are
(1) energy and y-component of linear momentum
(2) energy and z and y-components of linear momentum
(3) z and y-component of linear and x-component of angular momentum
(4) energy only

28. consider the system given below



The number of generalized coordinates required to describe the motion is:
(1) 2 (2) 3 (3) 5 (4) 6

29. A solid cylinder of radius a is rolling on the rough inside surface of a fixed cylinder of radius b>a. The time period of small oscillations about equilibrium position is:

(1) $2\pi\sqrt{\frac{3(b-a)}{2g}}$ (2) $2\pi\sqrt{\frac{2(b-a)}{3g}}$ (3) $2\pi\sqrt{\frac{b-a}{g}}$ (4) $\pi\sqrt{\frac{b}{g}}$